

# B.Sc. I

## MATHEMATICS

### Paper I

#### Algebra

Mapping, Equivalence Relations and partitions, Congruence Modulo  $n$ , Symmetric, Skew symmetric, Hermitian and Skew Hermitian Matrices, Elementary Operations on Matrices, Inverse of a matrix. Linear independence of row and column matrices. Row rank, Column Rank and Rank of a matrix. Equivalence of Column and Row Ranks. Eigen values, eigen vectors and the characteristic equation of a matrix. Cayley Hamilton Theorem and its use in finding inverse of a matrix. Application of matrices to a system of linear (both homogeneous and non-homogeneous) equations. Theorems on consistency of a system of Linear equations. Relations between the roots and coefficients of a general polynomial equation in one variable. Transformation of equations. Descartes's Rule of Signs. Solution of Cubic equations (Cardan's Method), biquadratic equations. Definition of a Group with examples and simple properties. Subgroup. Generation of Groups. Cyclic Groups. Coset Decomposition. Lagrange's Theorem and its consequences. Fermat's and Euler's Theorems. Homomorphism and Isomorphism. Normal Subgroups. Quotient Groups. The Fundamental Theorem of Homomorphism. Permutation Groups. Even and Odd Permutations. The Alternating Groups and Cayley's Theorem. Automorphism, Inner automorphism, Automorphism Groups and their compositions. Conjugacy Relation. Normaliser. Counting Principle and the Class equation of a finite group. Centre for Group of prime order. Abelianising of a Group and its universal property. Sylow's Theorems.  $p$ -Sylow Group. Structure Theorem for finite abelian groups. Introduction to Rings, Subrings, Integral Domain and Fields. Characteristic of a Ring. Trigonometry De Moivre's Theorem and its applications. Direct and Inverse Circular and Hyperbolic functions. Logarithm of a complex quantity. Expansion of Trigonometrical Functions. Gregory's Series. Summation of Series.

### Paper II

#### Differential Calculus

$\epsilon$ - $\delta$  definition of the limit of a function. Basic properties of limits. Continuous functions and classification of discontinuities. Differentiability. Successive Differentiation. Leibnitz's Theorem. Maclaurin's and Taylor's series expansion. Asymptotes. Curvature. Test for Concavity and Convexity. Points of Inflexion. Multiple Points. Tracing of Curves in Cartesian and Polar Coordinates.

#### Integral Calculus

Integration of irrational algebraic functions and transcendental functions. Reduction Formulae. Definite Integrals. Quadrature. Rectification. Volumes and Surfaces of solids of revolution.

#### Ordinary Differential Equations

Degree and Order of a differential equation. Equations of first order and first degree. Equations in which the variables are separable. Homogeneous equations. Linear equations and the equations reducible to Linear Form. Exact differential Equation. First Order higher degree equations solvable for  $x$ ,  $y$ ,  $p$ . Clairaut's Form and Singular Solutions. Geometric Meanings of a differential equation. Orthogonal Trajectories. Linear Differential Equations with Constant Coefficients. Homogeneous Linear ordinary differential equation. Linear Differential Equations of the Second Order. Transformation of the equation by changing the dependent variable/ independent variable. Method of variation of parameters. Ordinary Simultaneous Differential Equations.

### Paper III

#### Vector Analysis & Geometry

Scalar and Vector product of three vectors. Product of Four vectors. Reciprocal vectors. Vector Differentiation. Gradient, Divergence and Curl. Vector Integration. Theorems of Gauss, Green, Stokes and problems based on these. General equation of Second Degree. Tracing of Conics. System of Conics. Confocal Conics. Polar equation of a Conic. Plane. The straight line and the plane. Sphere. Cone. Cylinder. Central Conicoids. Paraboloids. Plane Sections of Conicoids. Generating Lines. Confocal Conicoids. Reduction of Second Degree Equations.

## B.Sc. II

### Paper – I

#### Advance Calculus

Continuity , Sequential Continuity , Properties of continuous functions , Uniform Continuity, Chain Rule of Differentiability. Mean Value Theorems and their geometrical interpretations. Darboux's Intermediate Value Theorem for derivatives. Taylor's Theorem with various forms of remainders. ( 10 Marks ) Limit and Continuity of functions of two variables. Partial Differentiations, Change of Variables. Euler's Theorem on Homogeneous functions. Taylor's Theorem for functions of two variables. Jacobians. ( 10 Marks ) Envelopes , Evolutes , Maxima Minima and Saddle points of functions of two variables. Lagranges Multiplier Method, Indeterminate Forms. ( 10 Marks ) Beta and Gamma Functions. Double and Triple integrals, Dirichlet's Integrals, Change of Order of integration in Double integrals. ( 10 Marks ) Definition of a sequence . Theorems on limits of Sequences. Bounded and Monotonic Sequences. Cauchy's Convergence Criterion. Series of Non - negative terms. Comparison Test, Cauchy's Root Test , Ratio Test , Raabe's Test , Logarithmic Test , DeMorgan's and Bertrand's Test , Alternating Series. Leibnitz's Theorem. Absolute and Conditional Convergence. ( 10 Marks )

### PAPER – II

#### ( Differential Equation )

Series Solution of differential equations, Power Series Method, Bessel and Legendre and Hypergeometric equations. Bessel and Legendre and Hypergeometric Functions and their properties, Recurrence and Generating Relations, Orthogonality of Functions , Sturm Liouville Problem. Orthogonality of Eigen Functions. Reality of Eigen values. Orthogonality of Bessel's Functions and Legendre's Polynomials. ( 10 Marks ) Laplace Transformation, Linearity of the Laplace Transformation. Existence theorem of Laplace Transformation. Laplace Transforms of Derivatives and Integrals. Shifting Theorems. Differentiation and Integration of transforms. Convolution Theorem. Solution of Integral Equations and System of differential equations using Laplace Transfor. ( 10 Marks ) Partial Differential equations of the first order. Lagrange's Solution. Some Special types of equations which can be solved easily by methods other than the general method. Charpit's General Method of solution. Partial differential equations of Second and higher orders. Classifications of Linear Partial Differential equations of second order. Homogeneous and Non homogeneous equations with constant coefficients. Partial differential equations reducible to equations with constant coefficients .Monge's Methods. ( 10 Marks ) Calculus of Variations, Variational Problems with fixed boundaries. Euler's Equation for functionals containing first order derivative and one independent variable. Extremals. Functionals dependent on higher order derivatives. Functionals dependent on more than one independent variable. Variational problems in parametric form. Invariance of Euler's equation under coordinates transformation. ( 10 Marks ) Variational Problems with Moving Boundaries, Functionals dependent on one and two functions. One sided variations. Sufficient conditions for an Extremum , Jacobi and Legendre Conditions. Second Variation Principle of Least action. ( 10 Marks )

### PAPER – III

#### ( Mechanics )

Statistics : Analytical conditions of equilibrium of Coplanar forces, Virtual Work , Catenary. Forces in three dimensions, Poincot's Central Axis, Wrenches , Null Lines and Planes, Stable and unstable Equilibrium. ( 20 Marks ) Dynamics : Velocities and Accelarations along radial and transverse directions and along tangential and Normal directions. Simple Harmonic Motion. Elastic Strings. Motion on smooth and rough plane curves, Motion in a resisting medium, Motion of particle of

varying mass. Central Orbits , Kepler's Laws of Motion. Motion of particle in three dimendions,  
Accelaration in terms of different coordinate systems. ( 30 Marks )

## B.Sc.III

### PAPER – I ( Analysis )

Real Analysis : Riemann Integral, Integrability of continuous and monotonic functions. The Fundamental Theorem of Integral Calculus. Mean Value Theorems of Integral Calculus. Improper Integrals and their convergence, Comparison Test, Abel's and Dirichlet's Tests. Frullani's Integral. Integral as the sum of a parameter. Continuity, Derivability and integrability of an integral of a function of a parameter. Series of arbitrary terms. Convergence, Divergence and Oscillation. Abel's and Dirichlet's Tests. Multiplication of series. Double Series. Partial Derivation and Differentiability of real valued functions of two variables. Schwartz's and Young's Theorem. Implicit Function Theorem. Fourier Series. Fourier expansion of Piecewise Monotonic Functions. ( 25 Marks )

Complex Analysis : Complex Numbers as Ordered Pairs. Geometric Representation of Complex Numbers. Stereographic Projection. Continuity and Differentiability of Complex Functions, Analytic Functions, Cauchy Riemann's Equations. Harmonic Functions. Elementary Functions. Mapping by elementary functions. Mobius Transformations. Fixed Points. Cross Ratio. Inverse Points and Critical Mappings. Conformal Mappings. ( 30 Marks )

Metric Spaces : Definition and examples of Metric Spaces. Neighbourhoods. Limit Points. Interior Points . Open and Closed Sets. Closure and Interior. Boundary Points. Subspace of a Metric Space. Cauchy's Sequence. Cantor's Intersection Theorem. Contraction Principle. Construction of Real Numbers as the Completion of the Incomplete Metric Space of rationals. Real numbers as a Complete Ordered Field. Dense subsets. Baire Category Theorem. Separable, Second Countable and First Countable Spaces. Continuous Functions. Extension Theorem. Uniform Continuity. Isometry and homomorphism. Equivalent Metrics. Compactness. Sequential Compactness. Totally Bounded Spaces. Finite Intersection Property. Continuous functions and Compact sets. Connectedness. Components. Continuous functions and connected sets. ( 20 Marks )

### Paper– II

#### ( Numerical Analysis )

Solution of Equations : Bisection, Secant, Regula Falsi, Newton's Method, Roots of Polynomials. Interpolation : Lagrange and Hermite Interpolation. Divided Differences. Difference Schemes. Interpolation Formula using Differences. Numerical Differentiation. Numerical Quadrature. Newton-Cotes Formulas. Gauss Quadrature Formula, Chebyshev's Formulas. Linear Equations. Direct Methods for solving systems of linear equations ( Gauss Elimination. LU decomposition, Cholesky Decomposition ), Iterative Methods ( Jacobi, Gauss Seidel, Relaxation Methods ) The Algebraic Eigenvalue Problem. Jacobi's Method, Given's Method, Householder's Method, Power Method, QR Method, Lanczos' Method. Ordinary Differential Equations. Euler Method, Single Step Methods, Runge-Kutta's Method, Multi Step Methods, Milne Simpson's Method, Methods based on Numerical Integration, Methods based on Numerical Differentiation, Boundary Value Problems, Eigenvalue Problems. Approximation. Different types of approximations, Least Square Polynomial approximation, Polynomial approximation using Orthogonal polynomials, Approximation with Trigonometric Functions. Exponential functions, Chebyshev's Polynomial, Rational Functions. ( 55 Marks )

Monte Carlo Methods : Random Number Generation, Congruential Generators, Statistical Tests of Pseudo Random Numbers. Random Variate Generation, Inverse Transform Method, Composition Method, Acceptance-Rejection Method, Generation of exponential, Normal Variates, Binomial and Poisson Variates. Monte Carlo Integration. Hit or Miss Monte Carlo Integration, Monte Carlo Integration for Improper Integrals. Error Analysis for Monte Carlo Integration. ( 20 Marks )

### Paper- III ( OPTIONAL )

#### 1. Differential Geometry

Local Theory of Curves : Space Curves, Examples, Planar Curves. Helices. Serret-Frenet Apparatus. Existence of Space Curves. Involutives and Evolutes of Curves. Global Theory of Curves : Rotation Index. Convex Curves. Isoperimetric Inequality. Four Vertex Theorem. Local Theory of Surfaces : Parametric Patches on Surface. First Fundamental Form and Arc Length. Normal Curvature. Geodesic Curvature and Gauss Formulae. Shape operator  $L_p$  of a surface at a point. Vector Field along a curve. Second and Third Fundamental Forms of the Surfaces. Weingarten Map. Principal Curvatures. Gaussian Curvature. Mean and Normal Curvatures. Gauss Theorem egregium. Isometry Groups and the Fundamental Existence theorem of surfaces. ( 15 Marks )

Global Theory of Surfaces : Geodesic Coordinate Patches. Gauss Bonnet Formulae. Euler-Characteristic of a Surface. Index of a Vector Field. Spaces of Constant Curvature. ( 10 Marks )

Intrinsic Theory of Surfaces in Riemannian Geometry. Parallel Translation and Connection. Cartan's Structural Equations and Curvature. Interpretation of Curvature. Geodesic Curvature and Gauss Bonnet for a 2 -Dimensional Riemann Surface. Geodesic Coordinate Systems. Isometries and Spaces of constant curvature and the 3-types of geometry. ( 10 Marks ) Transitive Extension Theory of Surfaces in  $R^3$ - Spherical image. Parallel translation for imbedded surfaces in  $R^3$ . Classification of Compact Connected Oriented surfaces in  $R^3$  relative to curvature. (10 Marks ) Elements of General Riemannian Geometry. Concepts of Manifolds and examples. Riemannian Metric. Tensor Fields Covariant Differentiation. Symmetry properties of Curvature Tensor. Concept of Affine Connection. Christoffel's Symbols. Curvature and Torsion Tensors. Riemannian Metric and Affine Connection Geodesic and Normal coordinates. Fundamental Theorem of Riemannian Geometry. ( 15 Marks )

### Paper- III ( OPTIONAL )

#### 2. Discrete Mathematics

Sets and Propositions : Cardinality. Mathematical Induction. Principle of Inclusion and Exclusion. Computability and Formal Languages . Ordered Sets . Languages. Phrase Structure Grammars. Types of Grammars and Languages. ( 15 Marks ) Permutations. Combinations and Discrete Probability. Relations and Functions. Binary Relations. Equivalence Relations and Partitions. Partial Ordered Relations and Lattices. Chains and Antichains. Pigeon Hole principle. ( 15 Marks )

Graphs and Planar Graphs. Basic Terminology. Multigraphs. Weighted Graphs. Paths and Circuits. Shortest Paths and Circuits. Travelling Salesman Problem. Planar Graphs. Trees. ( 15 Marks )

Finite State Machines. Equivalent Machines. Finite State Machines as Language Recognizers. Analysis of Algorithms. Time Complexity. Complexity of Problems Discrete Numeric Functions and Generating Functions. Recurrence Relations and Recursive Algorithms. Linear Recurrence Relations with constant coefficients. Homogeneous solutions. Particular Solution. Total Solution. Solution by Method of Generating Functions. ( 15 Marks )

Brief Review of Groups and Rings Boolean Algebras. Lattices and Algebraic Structures. Duality. Distributive and Complemented Lattices. Boolean Lattices and Boolean Algebras. Boolean Functions and Expressions. Propositional Calculus. Design and Implementation of Digital Networks. Switching Circuits. ( 15 Marks )

### Paper- III ( OPTIONAL )

#### 3. Mechanics

Dynamics of Rigid Bodies : Moments and Product of Inertia. The Momental Ellipsoid. Equipmomental Systems. Principal Axes D'Alembert's Principle. The General Equations of Motion of a rigid body. Motion of the Centre of Inertia and Motion relative to the Centre of Inertia. Motion

about a Fixed Axis. The Compound Pendulum. Centre of Percussion. Motion of a rigid body in Two Dimensions under finite and impulsive forces. Conservation of Momentum and Energy .Lagrange's Equations. Initial Motions. ( 40 Marks ) Hydrostatics : Pressure Equation. Condition of Equilibrium. Lines of Forces. Homogeneous and Hetrogeneous Fluids. Elastic Fluids. Surface of Equal Pressure. Fluid at rest under action of gravity. Rotating Fluids. Fluid Pressure on Plane Surfaces. Centre of Pressure. Resultant Pressure on Curved Surfaces. Equilibrium of Floating Bodies. Curves of Byoancy. Surface of Byoancy. Stability of Equilibrium of Floating bodies. Meta Centre. Work Done in producing a displacement. Vessel containing Liquid. Gas Laws. Mixture of Gases. Internal Energy. Adiabatic Expansion. Work Done in compressing a gas. Isothermal Atmosphere. Connective Equilibrium. ( 35 Marks )

**Paper – III ( OPTIONAL )**

#### **4. Mathematical Modelling**

The Process of Applied Mathematics. Setting up First order Differential equations. Qualitative Solution Sketching. Difference and Differential Equation Growth Models. Single Species Population Models. Population Growth. An age structure Model. The spread of Technological Innovation. ( 15 Marks ) Higher Order Linear Models. A model of Detection of Diabetes. Combat Modes. ( 12 Marks ) Traffic Models. Car following Models. Equilibrium Speed Distributions. ( 12 Marks ) Non Linear Population Growth Models. Prey-Predator Models. Epidemic Growth Models. ( 12 Marks ) Models from Political Science. Proportional Representation. Cumulative Voting. Comparison Voting. ( 12 Marks ) Application in Ecological and Environmental Subject ares. Urban

**Paper – III ( OPTIONAL )**

#### **5. APPLICATION OF MATHEMATICS IN FINANCE & INSURANCE**

Application of Mathematics in Finance\ Financial Management : An Overview. Nature and Scope of Financial Management. Goals of Financial Management and Main Decisions of Financial Management. Difference between Risk , Speculation and Gambling. Time Value of Money. Interest Rate and Discount Rate. Present Value and Future Value .Discrete case as well as continuous compounding case. Annuities and its kinds. Meaning of Return. Return as Internal Rate of Return ( IRR ). Numerical Methods like Newton Raphson Method to calculate IRR. Measurement of Returns under Uncertainty situations. Meaning of Risk. Difference between Risk and Uncertainty. Types of Risks. Measurement of Risk. Calculation of Security and Portfolio Risk and Return-Markowitz Model. Sharp's Single Index Model Systematic Risk and Unsystematic Risk. Taylor Series and Bond valuation. Calculation of Duration and Convexity of Bonds. Financial Derivatives – Future , Forward, Swaps and Options. Call and Put Options. Call and Put Parity Theorem. Pricing of contingent claims through Arbitrage and Arbitrage Theorem. ( 40 Marks ) Application of Mathematics in Insurance Insurance Fundamentals. Insurance defined. Meaning of Loss. Chances of Loss , Peril , Hazard and Proximate cause in insurance. Costs and Benifits of insurance to the society and branches of insurance. Life Insurance and various types of general Insurance. Insurable Loss Exposures. Feature of a Loss that is Ideal for Insurance. Life Insurance Mathematics. Construction of Mortality Tables. Computation of Premium of Life Insurance for a fixed duration and for the whole life. Determination of Claims for General Insurance. Using Poisson Distribution and Negative Binomial Distribution. The Polya Case. Determination of the amount of the claim in General Insurance. Compound Aggregate claim model and its properties and claims of Reinsurance. Calculation of a Compound Claim Density Function. F-Recursive and approximate formulae for F. ( 35 Marks )

### PAPER – III ( OPTIONAL )

#### 6. Special Theory of Relativity

Review of Newtonian Mechanics. Inertial Frames. Speed of Light and Galilean Relativity. Michelson Morley Experiment. Lorentz Fitzgerald Contraction Hypothesis. Relative character of space and time. Postulates of special theory of relativity. Lorentz Transformation equations and its Geometrical interpretation. Group properties of Lorentz transformations. ( 15 Marks ) Relativistic Kinematics. Composition of Parallel Velocities. Length Contraction. Time Dilation. Transformation equations for components of velocity and acceleration of a particle and Lorentz Contraction Factor. ( 15 Marks ) Geometrical Representation of Space time. Four Dimensional Minkowskian Space Time of Special Relativity. Time -Like, Light Like and Space Like Intervals. Null Cone. Proper Time. World Line of a particle. Four Vectors and Tensors in Minkowskian Space Time. ( 15 Marks ) Relativistic Mechanics. Variation of Mass with velocity. Equivalence of Mass and Energy. Transformation Equations for Mass Momentum and Energy. Energy-Momentum Four Vector. Relativistic Force and Transformation Equations for its Components. Relativistic Lagrangian and Hamiltonian. Relativistic Equations of motion of a particle. Energy Momentum Tensor of a Continuous Material Distribution. ( 15 Marks ) Electromagnetism. Maxwell's Equations in Vacuo. Transformation Equations for the densities of electric charge and current. Propagation of electric and magnetic field strengths. Transformation equations for Electromagnetic four potential vector. Gauge Transformation Equations for electric and magnetic field strengths. Gauge Transformation. Lorentz invariance of Maxwell's Equations. Maxwell's equations in Tensor Form. Lorentz Force on a charged particle. Energy Momentum Tensor of an Electromagnetic Field. ( 15 Marks )